University of California, Santa Cruz
Department of Applied Mathematics and Statistics
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Classical and Bayesian Inference - AMS 132

Review and Example

Suppose that $X_1, \ldots, X_n$ are random variables describing the number of problems that n students answer in a specific assignment. Assume that, potentially, the assignment can have infinite exercises. Assume that $X_1, \ldots, X_n$ form a random sample from the Poisson distribution with unknown parameter θ, where $\theta > 0$. A sample of 74 students was observed and the number of problems that all of the solved was 192. For the teacher it is of interest to estimate the mean and coefficient of variation of the numbers of exercises that students solved in their assignment.

If X is a r.v. with Poisson distribution with parameter $\theta > 0$, then $f(x \mid \theta) = \frac{e^{-\theta}\theta^x}{x!}$, $x = 0, 1, \ldots,$ $E(X) = \theta$, $Var(X) = \theta$.

if X is a random variable with gamma distribution with parameters $a > 0$, and $b > 0$, then $f(x \mid \theta) = \frac{b^a}{\Gamma(a)} x^{a-1} e^{-bx}$, $x > 0$, $E(X) = \frac{a}{b}$, $Var(X) = \frac{a}{b^2}$.

1. Write the statistical model.
2. Identify some statistical inference in the above problem.
3. Find the maximum likelihood estimates of interest.
4. Suppose that θ is a random parameter that has a gamma prior distribution that has mean and variance equal to 2. Find the Bayes estimates of interest under square error loss.
5. Find the distribution of a new observation given that $X_1 = x_1, \ldots, X_n = x_n$ have been observed.